B.Sc II part Paper IV

Differential Equation (Singular Solution)

Let the diff. Eqn. be denoted by $f(x,y,p)=0$; $p=\frac{dy}{dx}$ and its general Solution by $\phi(x,y,c)=0$ Where c is an arbitrary constant. The existence of Singular solution depends upon the p-discriminant of $f(x,y,p)=0$ and the c-discriminant of $\phi(x,y,c)=0$. Hence prior to embarking on the definition and existence of Singular Solution, we look at the discriminant of an equation whether ~~algebraic~~ algebraic or differential.

The discriminant of a quadratic eqn $ax^2+bx+c=0$ is b^2-4ac. The vanishing of the discriminant i.e $b^2-4ac=0$ express the condition that the two roots of the given eqn. are equal. i.e two values of x are equal. Similarly if $f(x)=0$ be any eqn. of higher order than two, then as we know from Theory of Equations that the condition for equal roots is obtained by eliminating x between $f(x)=0$ and $f'(x)=0$.

Let $f(x,y,p)=0$ be a differential eqn. and let its solution be $\phi(x,y,c)=0$ where c is an arbitrary const. Then the c-discriminant is obtained by eliminating c between $\phi(x,y,c)=0$ and $\frac{\partial\phi}{\partial c}=0$; treating x, y as constants and its vanishing expresses the

Condition that the eqn. $\phi(x,y,c)=0$ regarded as an equation in c, has equal values c.

Hence we can say that the c-discriminant of $\phi(x,y,c)=0$ represents the locus for each point of which $\phi(x,y,c)=0$ has equal values of c.

Again, if $f(x,y,p)=0$ be the differential equation whose general solution is $\phi(x,y,c)=0$, then the p-discriminant is obtained by eliminating p between $f(x,y,p)=0$ and $\frac{\partial f}{\partial p}=0$, treating x, y as constants and its vanishing expresses the condition that the equation $f(x,y,p)=0$ regarded as an equation in p has equal values of p.

Hence, we can say that the p-discriminant of $f(x,y,p)=0$ represents the locus for each point of which $f(x,y,p)=0$ has equal values of p.

If $y=px+f(p)$; $p=\frac{dy}{dx}$ ——— (1)

be a diff. eqn. in Clairaut's form, then its general solution is $y=cx+f(c)$ ——— (2)

where c is an arbitrary constant. Obviously the p-discriminant of (1) and c-discriminant of (2) are the same and that gives us what is called the singular solution of the D.E (1).

However, if $f(x,y,p)=0$ (3) be any diff. eqn. not necessary in Clairaut's form and if

$\phi(x,y,c)=0$ ——— (4) be its solution, then are we suppose that p-discriminant of (3) and c-discriminant of (4) are the same? Obviously not.